Solution by G. B. M. ZERR. A. M., Ph. D., Philadelphia, Pa.

Let $y^2=4a(x+a)$, $(y\cos\beta+x\sin\beta)^2=4A(x\cos\beta-y\sin\beta+A)$ be the confocal parabolas.

(1) $y^2-4ax-4a^2=0$.
(2) $y^2\cos^2\beta+x^2\sin^2\beta+2xy\sin\beta\cos\beta-4Ax\cos\beta+4Ay\sin\beta-4A^2=0$.

Calculating the invariants for (1) and (2) we get

$$\triangle=-4a^2,\quad \Theta=-4a(a+2A\cos\beta),$$
$$\Theta'=-4A(A+2a\cos\beta),\quad \triangle'=-4A^2.$$

The condition is given by $\Theta^2=4\triangle\Theta'$.

$\therefore 16a^2(a+2A\cos\beta)^2=16a^2A(A+2a\cos\beta)$.

$\therefore a^2/A^2+2(a/A)\cos\beta+4\cos^2\beta=1$, $a/A=-\cos\beta\pm\sqrt{(1-3\cos^2\beta)}$.

$\therefore \cos\beta>1/\sqrt{3}$.

$\therefore\beta$ lies between 54° 44′ and 125° 16′, and also between 234° 44′ and 305° 16′.

If $\beta=\frac{1}{3}\pi$, $a=\infty A$ or $-A$.

If $\beta=\frac{1}{2}\pi$, $a=A$ or $-A$.

If $\beta=\frac{2}{3}\pi$, $a=A$ or $\infty A$, etc.

## CALCULUS.

282. Proposed by S. G. BARTON, Ph. D., Clarkson School of Technology, Potsdam, N. Y.

A rectangular beam of length $l$ and width $w$ is taken horizontally from a hall of width $b$ into a corridor at right angles to the hall. Find the width of the smallest corridor into which it can be taken.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa., and J. M. ARNOLD, Crompton, R. I.

Let $ABCD$ be the beam, the corner $A$ against the hall wall, the corner $B$ against the corrider wall; the point $P$ of the beam against the corner of of meeting of hall and corrider.

Let $AB=l$, $BC=w$, $x=$width of corrider, $PQ$ the portion of the width the corrider under the beam, $QR$ the remainder of the width.

Then $PQ+QR=x$, $AQ+QB=l$, $ER=b$, $\angle BAE=\theta$. Then $PQ=w\operatorname{cosec}\theta$, $AQ=b\operatorname{cosec}\theta$, $QR=(l-b\operatorname{cosec}\theta)\cos\theta$.

$\therefore x=w\operatorname{cosec}\theta+(l-b\operatorname{cosec}\theta)\cos\theta\ldots(1)$.

Differentiating (1) we get, $w\operatorname{cosec}\theta\cot\theta+l\sin\theta=b\operatorname{cosec}^2\theta$.

$\therefore w\cos\theta=b-l\sin^3\theta\ldots(2)$.

The value of $\theta$ from (2) in (1) gives the width $x$ required.

(2) becomes $l^2\sin^6\theta-2bl\sin^3\theta+w^2\sin^2\theta+b^2-w^2=0$.

Also solved by J. E. Sanders and the Proposer.